
Annex 2: The Gelfand Levitan Marchenko method

Define \hat{r} the Fourier transform of r :

$$\hat{r}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} r(\beta) \exp(-i\beta\tau) d\beta \quad \tau \in \mathbf{R}$$

The GLM methods leads to the following integral equation for \hat{r} :

$$\hat{r}(\tau) = -\frac{1}{2}q\left(\frac{\tau}{2}\right) - \int_0^{\frac{\tau}{2}} dz_1 \int_0^{z_1} dz_2 q(z_1)\bar{q}(z_2)\hat{r}\left(\frac{\tau}{2} - (z_1 - z_2)\right)$$

The iterative algorithm is then the following:

$$\begin{cases} -\frac{1}{2}q^0\left(\frac{\tau}{2}\right) = \hat{r}(\tau) \\ -\frac{1}{2}q^{n+1}\left(\frac{\tau}{2}\right) = \hat{r}(\tau) + \int_0^{\frac{\tau}{2}} dz_1 \int_0^{z_1} dz_2 q^n(z_1)\bar{q}^n(z_2)\hat{r}\left(\frac{\tau}{2} - (z_1 - z_2)\right) \end{cases}$$