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A fully adaptive hybrid optimization of aircraft engine blades

L. Dumas [a,*], B. Druez [b], N. Lecerf [b]

[a] Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie, 75252 Paris Cedex 05, France
[b] Snecma moteurs (groupe Safran), site Villaroche, France

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ABSTRACT

A new fully adaptive hybrid optimization method (AHM) has been developed and applied to an industrial problem in the field of the aircraft engine industry. The adaptivity of the coupling between a global search by a population-based method (Genetic Algorithms or Evolution Strategies) and the local search by a descent method has been particularly emphasized. On various analytical test cases, the AHM method overperforms the original global search method in terms of computational time and accuracy. The results obtained on the industrial case have also confirmed the interest of AHM for the design of new and original solutions in an affordable time.

1. Introduction

In order to minimize a given cost function J defined from $\mathcal{O} \subset \mathbb{R}^n$ to $\mathbb{R}$, population-based methods such as Genetic Algorithms [1], Evolution Strategies [2] or more recently Particle Swarm Optimization [3] are more and more used in a very large number of industrial fields.

However, all these methods that make a population of potential solutions evolve are very time consuming because of the large number of evaluations of the functional that is needed. The idea of coupling them with a gradient-based method for a more efficient local search, leading to what is called hybrid methods, has shown its efficiency in many situations in the last decade [4–10], for instance for aerodynamic shape optimization, either in 2D [4] or more recently in 3D configurations for cars [8,9]. Unfortunately, the introduction of new parameters in hybrid methods reduces their range of applications and makes them 'problem-dependant'.

In this paper, a fully adaptive hybrid method inspired from [7] and called AHM, is developed. Using no extra parameters compared with the original global search method, the AHM algorithm is presented in Section 2 and then successively applied on analytical test functions in Section 3 and on an industrial problem, the shape optimization of aircraft engine blades, in Section 4.

2. Adaptive hydrid optimization method

The general principles of global and local searches that will be used here are recalled in the first two subsections. The coupling algorithm between both of them is then presented in Section 2.3 leading to the so-called Adaptive Hybrid Method (AHM).

* Corresponding author.
E-mail addresses: dumas@ann.jussieu.fr (L. Dumas), bruno.druez@snecma.fr (B. Druez), nicolas.lecerf@snecma.fr (N. Lecerf).
URL: http://www.ann.jussieu.fr/~dumas (L. Dumas).

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2.1. Description of the global search

Two population-based methods have been tested, namely Genetic Algorithms and Evolution Strategies, as the core tool of the hybrid method. The main principle of both of them is recalled below.

2.1.1. Genetic Algorithms (GA)

Genetic Algorithms are global optimization methods directly inspired from the Darwinian theory of evolution of species [1]. They consist in following the evolution of a certain number N_p of possible solutions, also called populations. To each element (or individual) $x_i \in \mathcal{O}$ of the population is affected a fitness value inversely proportional to $J(x_i)$ in case of a minimization problem. The population is regenerated N_g times by using three stochastic principles called selection, crossover and mutation, that mimic the biological law of the 'survival of the fittest'.

The Genetic Algorithm that will be used here acts in the following way: at each generation, $\frac{N_p}{2}$ couples are selected by using a roulette wheel process with respective parts based on the fitness rank of each individual in the population. To each selected couple, the crossover and mutation principles are then successively applied with a respective probability p_c and p_m. The crossover of two elements consists in creating two new elements by doing a barycentric combination of them with random and independent coefficients in each coordinate. The mutation principle consists in replacing a member of the population by a new one randomly chosen in its neighborhood. A one-elitism principle is added in order to be sure to keep in the population the best element of the previous generation. Thus, the algorithm writes as:

- Choice of an initial population $P_1 = \{x_i^1 \in \mathcal{O}, 1 \le i \le N_p\}$
- $n_g = 1$. Repeat until $n_g = N_g$
- Evaluate $\{J(x_i^{n_g}), 1 \le i \le N_p\}$ and $m = \min\{J(x_i^{n_g}), 1 \le i \le N_p\}$
- 1-elitism: if $n_g \ge 2$ & $J(X_{n_g-1}) < m$ then $x_i^{n_g} = X_{n_g-1}$ for a random i
- for k from 1 to $\frac{N_p}{2}$
-     Selection of $(x_\alpha^{n_g}, x_\beta^{n_g})$ with a roulette wheel process
-     with probability p_c: replace $(x_\alpha^{n_g}, x_\beta^{n_g})$ by $(y_\alpha^{n_g}, y_\beta^{n_g})$ by crossover
-     with probability p_m: replace $(y_\alpha^{n_g}, y_\beta^{n_g})$ by $(z_\alpha^{n_g}, z_\beta^{n_g})$ by mutation
- end for
- $n_g = n_g + 1$.
- Generate the new population P_{n_g}.
- Call X_{n_g} the best element.

2.1.2. Evolution Strategies (ES)

Evolution Strategies (ES) have been first introduced by Schwefel in the 60's [2]. As it is the case for Genetic Algorithms, it consists in following the evolution of a population of potential solutions through the same three stochastic principles, selection, recombination and mutation. However, contrarily to Genetic Algorithms, the major process is the mutation process and the selection is made deterministic.

The Evolution Strategy that will be used here is based on the $(\mu+\lambda)$ selection principle and on the 1/5 rule for the mutation strength. The latter means that the mutation strength σ is readjusted with respect to the rate of successful mutations in a generation: if this rate is greater than 1/5, then σ is multiplied by a factor $\alpha > 1$, or else it is divided by the same factor α. An intermediate recombination with two parents is also included. The algorithm thus writes as:

- Choice of an initial population of μ parents: $P_1 = \{x_i^1 \in \mathcal{O}, 1 \le i \le \mu\}$
- $n_g = 1$. Repeat until $n_g = N_g$
- Creation of a population of $\lambda \ge \mu$ offsprings O_{n_g} by:
-     Recombination: $y_i^{n_g} = \frac{1}{2}(x_\alpha^{n_g} + x_\beta^{n_g})$
-     Normal mutation: $z_i^{n_g} = y_i^{n_g} + \mathcal{N}(0, \sigma)$
- Update of the mutation strength σ with the 1/5 rule of parameter α
- Evaluate $\{J(z_i^{n_g}), z_i^{n_g} \in O_{n_g}\}$
- $n_g = n_g + 1$
- Selection of the best μ new parents in the population $P_{n_g} \cup O_{n_g}$.
- Call X_{n_g} the best element.

2.2. Description of the local search

A first order steepest descent method with a backtracking line search strategy has been chosen. Note that the local search has voluntarily been chosen to be basic in order to emphasize the coupling gain. Actually, any more sophisticated descent method (such as Newton or quasi Newton methods) could have been used with the same conclusions about the coupling interest.

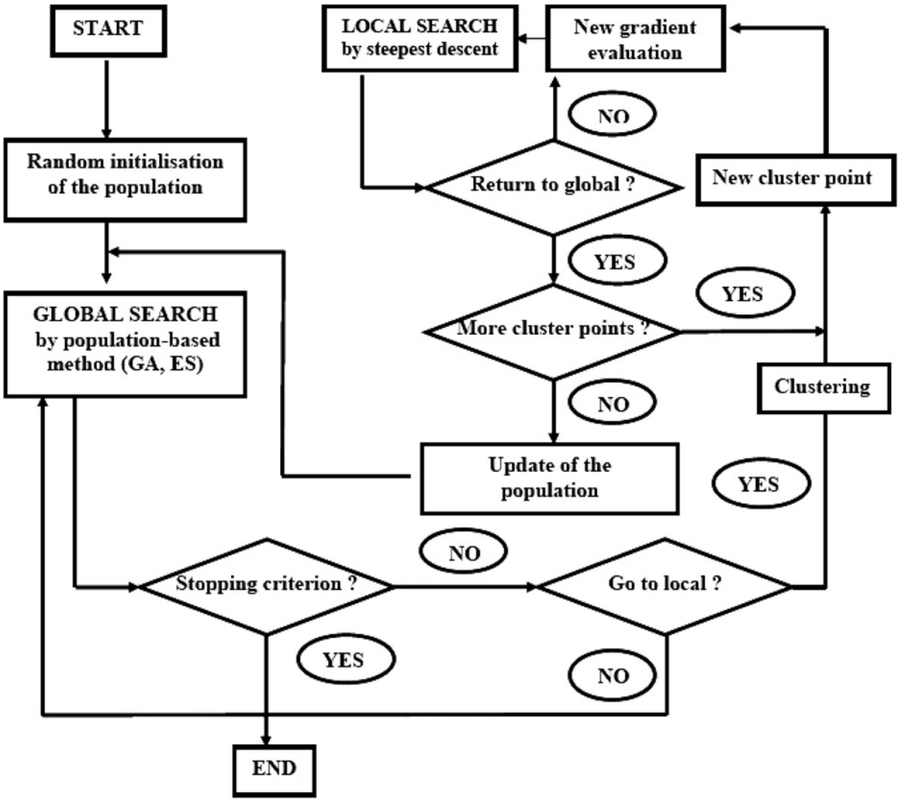

Fig. 1. General algorithm of AHM.

2.3. *Coupling principles*

2.3.1. *General algorithm*

In order to improve the convergence of population-based algorithms for time consuming applications, in other words to make the cost function decrease as much as possible for a given number of evaluations, the idea of coupling them with a deterministic local search has been explored for many years [4–10]. However, the obtained gain can be very different from one function to another, depending on the level of adaptivity of the coupling and the way to do it.

The present method, called Adaptive Hybrid Method (AHM) in which general principles are summarized in Fig. 1 try to remedy to these drawbacks by answering in a fully adaptive way to the three fundamental questions in the construction of a hybrid method:

- Question 1: when do we shift from the global search to the local search?
- Question 2: when do we come back to the global search?
- Question 3: on which elements do we apply a local search?

In order to answer to Questions 1 and 2, the AHM method uses some criteria introduced in the pioneer work of [7] whereas a new strategy called 'reduced clustering' allows to answer to Question 3 on a fully adaptive way.

2.3.2. *The shift from global to local (the answer to Question 1)*

The shift from a global search to a local search is useful when the exploration ability of the global search is no longer efficient. To this aim, a statistical coefficient associated with the cost function repartition values is introduced. It is equal to the ratio of the mean evaluation of the current population X to its corresponding standard deviation:

$$CV = \frac{m}{\sigma} = \frac{\text{mean}\{J(x),\ x \in X\}}{\sqrt{\text{Var}\{J(x),\ x \in X\}}} \tag{1}$$

and is named coefficient of variation. A local search will be called when this ratio is increasing between two consecutive generations of the global search.

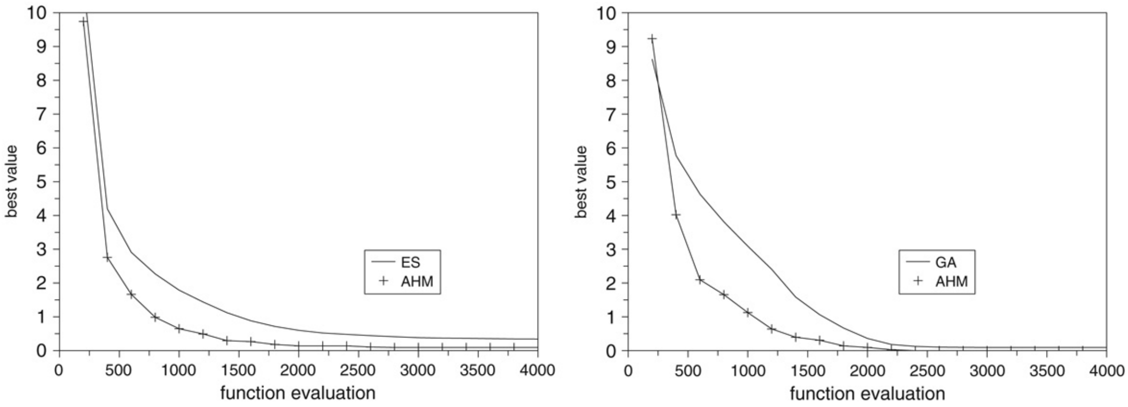

Fig. 2. Comparison of ES (left) and GA (right) with AHM on $Rast_1$, $n = 6$.

2.3.3. The shift from local to global (the answer to Question 2)

The local search, a gradient based method with a line search strategy, is aimed at making the cost function locally decrease more efficiently than the normal random mutation. However, this gain must be counterbalanced each time before evaluating a new gradient vector with a characteristic gain of the global method. More precisely, the local search will here continue while:

$$G_{local} > G_{global}$$

where G_{local} is equal to the gain when passing from a point to the next one in the steepest descent algorithm and G_{global} is the gain of the last global phase (evaluated with the decrease of m in formula (1)). Both gains are scaled with the number of evaluations of the cost function needed to achieve them.

2.3.4. The reduced clustering strategy (the answer to Question 3)

In order to spread as much as possible the local search in the whole domain, the population is divided into a certain number of sub-populations, called clusters. To do so, a very classical and fast algorithm is used where each cluster is constructed such that all its associated elements are closer to its center of mass than to any other. After this preliminary step called clustering, the local search is applied to the best element (with respect to J) of each cluster.

A careful study of the appropriate number of clusters has never been done even though it appears to be rather important for the algorithm's performance. To overcome the difficulty of choosing this number, we propose here a new method called reduced clustering where the number of clusters is progressively decreased during the optimization process. It corresponds to the natural idea that the whole process will progressively focus on a reduced number of local minima. To do so, a deterministic rule of arithmetic decrease plus an adaptive strategy including the aggregation of too near clusters has been used here. Note that the effect of the initial number of clusters has also been investigated in test functions. It appears that a initial number of clusters between 5% and 20% of the population will give the best results. Below 5%, some premature convergence can be observed whereas above 20%, the convergence speed may be reduced. The value of 10% will thus be adopted for all the simulations presented in the next section.

3. Adaptive Hybrid Method applied to test functions

Before applying it on a real industrial problem, the Adaptive Hybrid Method has first been tested on various analytical test functions, and among them the well known Rastrigin function of parameter a:

$$Rast_a(x_1, \dots, x_n) = \sum_{i=1}^{n} \left(x_i^2 - a\cos(2\pi x_i)\right) + an$$

defined on $\mathcal{O} = [-5, 5]^n$ for which there exists many local minima and only a global minimum located at $x^* = (0, \dots, 0)$. Note that the number of local minima in $\mathcal{O}$ is always equal to 11^n but the attraction basins become steeper when $a > 0$ takes a higher value.

A statistical comparison based on 40 independent runs for each method, has been made between a classical population based method, either a GA or an ES, and the associated AHM method. The result are displayed on Figs. 2–4. For each method, the average best obtained value is plotted with respect to the number of function evaluations needed to achieve it. Note that any gradient evaluation counts for n evaluations of the cost function as it is the case in many real applications for which the gradient has to be approximated by finite differences.

For the simulations presented below, the population size for the GA is either equal to $N_p = 40$ if $n = 6$ or $N_p = 100$ if $n = 20$ whereas $(\lambda, \mu) = (35, 5)$ for the ES. A maximal number of function evaluation is allowed, either equal to 4000 when $n = 6$ and 10 000 when $n = 20$.

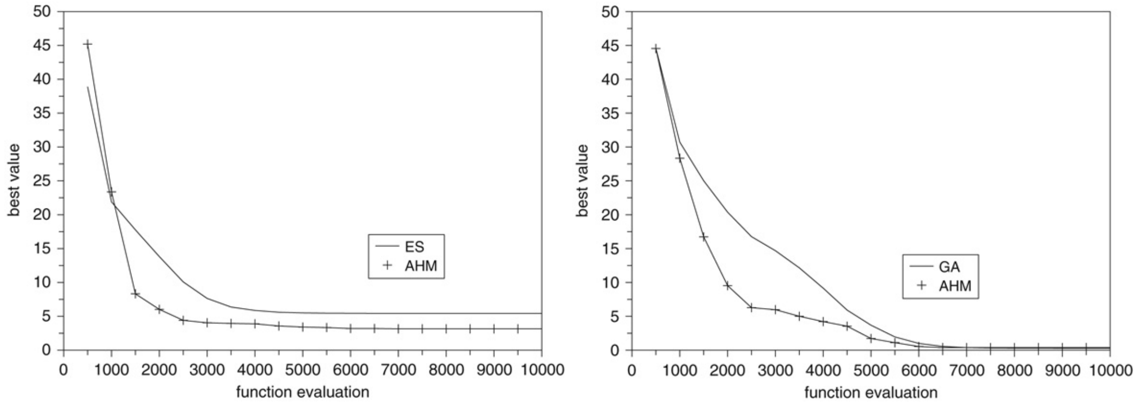

Fig. 3. Comparison of ES (left) and GA (right) with AHM on $Rast_1$, $n = 20$.

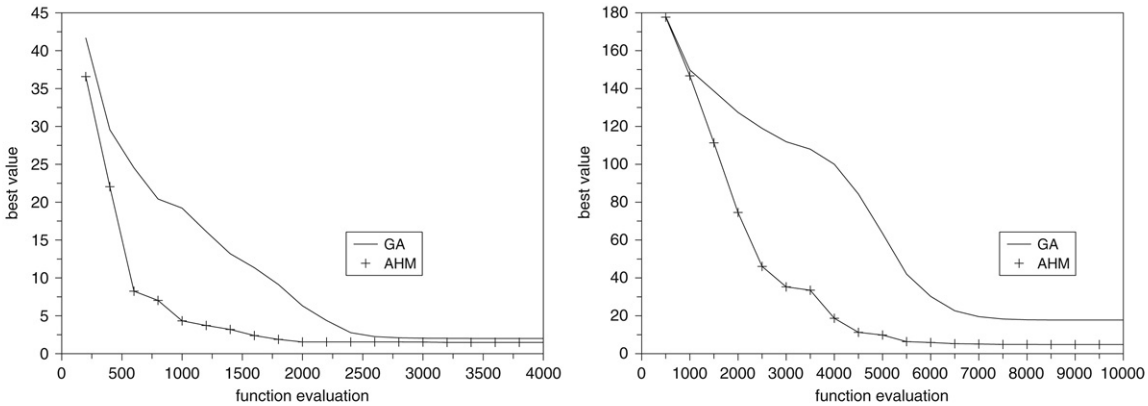

Fig. 4. Comparison of GA with AHM on $Rast_{10}$, $n = 6$ (left) and $n = 20$ (right).

Table 1
Comparison of GA and AHM after 5000 evaluations of $Rast_{10}$, $n = 20$.

Method	Mean obtained value	Best value	Worst value
GA	63.56	45.54	90.49
AHM	9.82	4.97	14.06

3.1. Result for the $Rast_1$ function

A comparison between the use of a GA or an ES as the global search method for the AHM method is made on the $Rast_1$ function with 6 and 20 parameters. The results, displayed on Figs. 2 and 3 respectively, show that the AHM method takes advantage compared with a pure ES or a pure GA in terms of the convergence speed to the optimal best value. In this situation, the GA results are better than the ES results. In particular, there are no longer cases of premature convergence to a local minima, particularly if $n = 20$. It implies that, in this case, the AHM method with a GA base performs better than the AHM with an ES base. However, it is necessary to be careful about any generalization of this observation.

3.2. Result for the $Rast_{10}$ function

Another example of comparison between a pure GA and the corresponding AHM for the $Rast_{10}$ function is visible on Fig. 4. A statistical comparison is also drawn on Table 1 where the mean, the best, and the worst value for GA and AHM are displayed after 5000 evaluations of the $Rast_{10}$ function with $n = 20$.

The conclusions are the same than for the $Rast_1$ function but with larger differences between both methods, always in the advantage of AHM. In other words, it means that the interest in using AHM increases with the difficulty of the problem (related here to the number of local minima).

Compared with other results found in the literature for the Rastrigin function (see for instance [10,11]), these results are very competitive. Moreover, it is worth noticing that the same general conclusions have been observed for other test functions similar to the Rastrigin function, that is with many local minima, namely the Ackley and the Griewank functions.

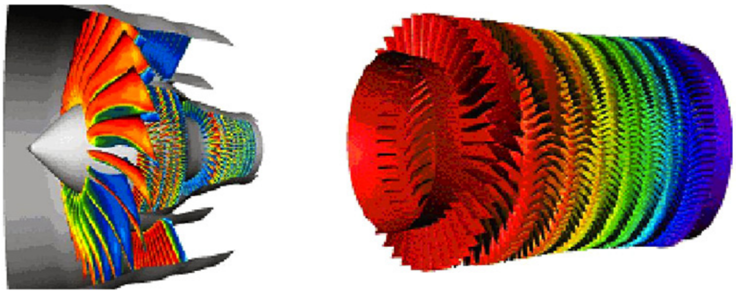

Fig. 5. Blades in the fan (left) and the high pressure compressor module (right).

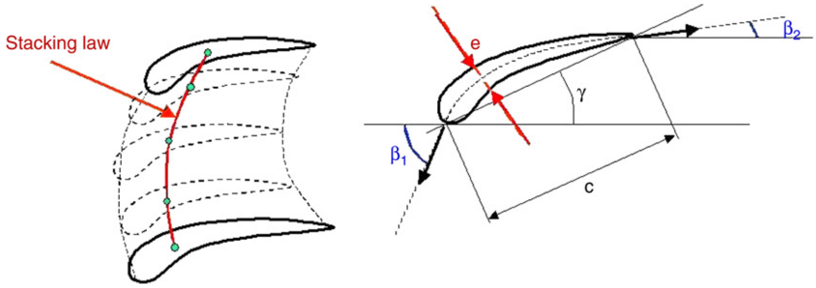

Fig. 6. Design parameters of a 3D blade.

At this point, the main conclusion that can be stated is that the Adaptive Hybrid Method will have two positive effects compared with a pure global search strategy (GA, ES, etc ...): at the early stage of the computation, it will enhance the decrease of the cost function and, at a late stage, it may also improve the final optimal solution. Such effects are particularly interesting in an industrial process where the computational time can be very large and also where any small improvement to the optimal solution can lead to significant productivity gains.

4. Adaptive Hybrid Method applied to aircraft engine blades optimization

4.1. General description of the optimization case

In a turboreactor, the blades, which represent a large amount of an engine price (nearly 35%), are designed in order to create and control the aerodynamic flow through the engine (see Fig. 5). As a first application of the Adaptive Hybrid Method previously described, the objective here is to optimize the design of the blades in the high pressure compressor module in order to minimize the mechanical efforts applied on them. Actually, it represents only a first step in the field of blade optimization as, for a high pressure compressor designer, increasing the isentropic efficiency of the compressor is the main target. Nevertheless, this goal cannot be achieved regardless other engine features. Among them is the stall margin. This aerodynamic instability phenomena consists of the stall of the flow around the blades. This leads to backward flow inside the compressor and can result in engine shutdown, overtemperature in the low pressure turbine, high level of vibration or blade out. To prevent such events, the designer will have to increase the compressor pressure ratio for low mass flow rates.

4.2. Details of the computation

A 3D blade can be broken down into a set of several 2D airfoil profiles. The different airfoils are linked to the original blade through the stacking law (see Fig. 6). Each airfoil can then be described by a set of design parameters which reflects physical phenomena that can be seized by the human designer. Fig. 6 shows some of common design parameters of the 2D profiles such as chord (c), maximum thickness value (e), upstream and downstream skeleton angles (β_1 and β_2), stagger angle (γ). In the presented case, these parameters are kept fixed whereas the parameters to optimize, of the number of six, are all associated with the stacking law.

In order to minimize the mechanical efforts on the blade, the associated function to minimize is equal to the maximal value on 2D profiles of the Von Mises constraints (see [12] for more details on this criteria). Such problem is highly non linear, has a large number of constraints, many local minima and is also time consuming. A global, fast and robust method, such as the AHM method, is thus needed.

Table 2
The blade optimization problem, performance comparison.

Optimization method	Number of evaluations	Best obtained value
Commercial code	460	163.5
AHM	480	158.6

4.3. Obtained results

The AHM method with a reduced clustering strategy has been used here to solve the blade optimization problem described above. Note in particular that constraints are handled with a penalization term whereas the gradients are approximated by finite differences.

The results are compared in Table 2 with those obtained with a commercial optimization platform based on a pure stochastic algorithm. It can be seen that for the same simulation time (approximately 80 hours CPU), the Adaptive Hybrid Method overperforms the commercial code. Indeed, even if the relative decrease obtained in the cost function appears to be small (3% approximately), it actually represents a significant improvement for the blade design. Note that such very interesting optimal value had never been previously achieved, even after many independent runs of the commercial code.

5. Conclusion

A fully adaptive hybrid optimization method (AHM) has been introduced and applied to a first applicative case in the field of aircraft engine blade optimization. The idea of coupling in a fully adaptive way a population based method (either Genetic Algorithm of Evolution Strategies) with a local search descent method appears to be very successful. Indeed, it allows a faster decrease of the cost function at the early stage of the process and it also improves the final optimal solution obtained. The main interest of the method is its independence to any additional parameter, which thus makes it applicable to a very large range of industrial problems.

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