Theory of Economic Growth and Taxation

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Introduction

Growth can be reached by the accumulation of capital and from innovations which lead to technical progress. Accumulation and innovation increase the productivity of inputs into production and provides the potential level of output [1].

The rate of growth can be affected by policy choices through the taxation [2]. An increase in taxation reduces the returns to investment. Lower returns mean less accumulation and innovation, and hence a lower rate of growth. This is the negative aspect of taxation. Indeed, some public expenditure can enhance productivity, such as the provision of infrastructure, public education, and health care.

F.Ramsey(1903-1930) is the founder of modern economic growth theory and optimal taxation. Ramsey [4] asked two dual questions:
1. For a given amount of revenue to be extracted from a consumer, what tax system makes the consumer happiest ?.
2. For a given level of happiness, how can we extract the most revenue from a consumer?

But Scully [6] [7] developed a model to estimate optimal tax burden rate showing that tax-growth relationship is inverse U-shaped. While some recent studies use quadratic method to find the optimal size of tax revenue. They clearly show that relationship is positive up to certain level and starts to become negative after crossing that level [8].

This research tries to estimate the growth-maximizing tax burden rate using the data from Mongolia economic statistics.

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Ramsey's Problem

Assumptions

1. *Hicksian demand curves are linear in the relevant range. For good i, the demand curve is:*

$$p_i = a_i - b_i x_i, \tag{3.1}$$

2. *The supply of good i is infinitely elastic at price c_i.*
3. *Lump sum tax are not available. In addition to the N goods under consideration, there is at least one other good that cannot be taxed.*
4. *Hicksian cross-price elasticities are zero*
5. *The required government spending G is fixed. It doesn't depend on the chosen mix of taxes.*

Ramsey's first problem is formulated as:

$$\min_{\tau_1,\tau_2,\ldots,\tau_N} \sum_{i=1}^{N} \frac{1}{2}\frac{c_i^2}{b_i}\tau_i^2, \tag{3.2}$$

subject to

$$\sum_{i=1}^{N} c_i\tau_i \frac{a_i - c_i - c_i\tau_i}{b_i} > G. \tag{3.3}$$

Taking the first order condition with respect to τ_i and solving for τ_i, we get

$$\tau_i = \frac{\lambda}{1+2\lambda}\frac{1}{\eta_i} \tag{3.4}$$

Ramsey's second problem is formulated as:

$$\max \sum_{i=1}^{m} R_i, \tag{3.5}$$

subject to

$$\sum_{i=1}^{m} u_i \geq K. \tag{3.6}$$

where, R_i is a tax revenue from i-th consumer, u_i is utility function, K is a given level of the utility.

There are three properties of Ramsey's solution.

1. Any good that can be taxed should be taxed to some degree.
2. Goods with higher demand elasticities should be taxed less.
3. the size of government affects all tax rates proportionally.

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$$Y = C + I + G,$$
$$C = \beta(1-\alpha)Y,$$
$$G = \alpha Y,$$
$$Y = F(K),$$
$$\frac{dK}{dt} = I + G,$$

where α is the tax rate, $(1-\alpha)Y$ is disposable income, G is the government revenue, $F(K)$ is a production function.

Maximization of total consumption over some horizon [0,T]:

$$\max_{\alpha} \int_0^T C(t)dt$$

subject to

$$K' = F(K) - \beta(1-\alpha)F(K)$$

$$K(0) = K_0,$$

where, K is capital.

Ramsey's optimal control formulation of growth theory:

$$\max \int_0^T U(C(t))dt$$

subject to

$$K' = F(K, L) - C - \mu K,$$

$$K(0) = K_0,$$

$$L(t) = L_0 e^{\gamma t},$$

$$K(t) \geq 0,$$

$$0 \leq C(t) \leq F(K(t), L(t)),$$

where, L is labour, μ is depreciation rate, γ is growth rate of population.

Solow growth theory with a taxation:

$$\max c = \frac{C}{L}$$

subject to

$$d(\frac{K}{L}) = 0$$

where, K is capital, L is labour, c is consumption per capita.

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Concavity Property of the Production Function

The production function we use is a type of the Gobb-Douglas form [6] [7]:

$$Y_t = \alpha_0 (G_{t-1})^{\alpha_1} [(1 - T) Y_{t-1}]^{\alpha_2}, \tag{5.1}$$

where, Y is the output, G is the government spending on public goods, T lamp sum tax for the time period t, α_1, α_2 are elasticity coefficients. The government budget requires that tax revenue equals the cost of public goods provided which means that:

$$G_t = TY_{t-1}, \tag{5.2}$$

where, T is the tax rate or a proportion of tax revenue in GDP.

Then combining (3.1) and (3.2), we can write

$$Y_t = \alpha_0 (TY_{t-1})^{\alpha_1} [(1 - T) Y_{t-1}]^{\alpha_2}, \tag{5.3}$$

Dividing both sides of the expression (3.3) by Y_{t-1}, we get

$$\frac{Y_t}{Y_{t-1}} = \alpha_0 T^{\alpha_1} (1 - T)^{\alpha_2} (Y_{t-1}^{\alpha_1 + \alpha_2 - 1}), \tag{5.4}$$

By definition of growth rate, we have $\frac{Y_t}{Y_{t-1}} = \theta$. Therefore, introducing a constant A as $A = \alpha_0 Y_{t-1}^{\alpha_1 + \alpha_2 - 1}$, we can write the function θ as

$$\theta = AT^{\alpha_1} (1 - T)^{\alpha_2} - 1. \tag{5.5}$$

Lemma

Under the assumption $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_1 + \alpha_2 \leq 1$ the function θ is concave.

Proof: The function θ can be presented a function of two variables x and y in $\theta = f(x, y)$, where $f(x, y) = Ax^{\alpha_1}y^{\alpha_2} - 1$, $x = T$ and $y = 1 - T$. In order to prove concavity of the function θ, we need to check a sign of its second order derivative.

$$\frac{\partial \theta}{\partial T} = \alpha_1 A x^{\alpha_1 - 1} y^{\alpha_2} - A\alpha_2 x^{\alpha_1} y^{\alpha_2 - 1},$$

$$\frac{\partial^2 \theta}{\partial^2 T} = Ax^{\alpha_1 - 2} y^{\alpha_2 - 2}[\alpha_1(\alpha_1 - 1)y^2 - 2\alpha_1\alpha_2 xy + \alpha_2(\alpha_2 - 1)x^2],$$

$\alpha_1(\alpha_1 - 1)y^2 - 2\alpha_1\alpha_2 xy + \alpha_2(\alpha_2 - 1)x^2 \leq 0$ which proves the concavity of θ.

Now we consider the problem of maximizing economic growth with respect to tax rate. This problem is $\max_T \theta$. From (3.5) we can get

$$\frac{\partial \theta}{\partial T} = A[\alpha_1 T^{\alpha_1 - 1}(1 - T)^{\alpha_2} - \alpha_2(1 - T)^{\alpha_2 - 1} T^{\alpha_1}] = 0$$

Hence, we have

$$T^* = \frac{\alpha_1}{\alpha_1 + \alpha_2} \tag{5.6}$$

Note that if assumption Lemma violates, in other words, if $\alpha_1 + \alpha_2 > 1$ then function θ may not be concave on $[0, 1]$, but T^* is still its maximum point.

It is easy to check that if $\alpha_1 + \alpha_2 = 1$ then the function $(\theta + 1)$ is constant returns to scale. Therefore, the production function (3.5) has the form

$$\theta + 1 = AT^{\alpha_1}(1 - T)^{\alpha_2}, \alpha_1 + \alpha_2 = 1.$$

Taking log from both sides, we obtain

$$\ln(\frac{1+\theta}{1-T}) = \ln(\alpha_0) + \alpha_1 \ln(\frac{T}{1-T}) \qquad (5.7)$$

$$\ln(Y_t) = \alpha_0 + \alpha_1 \ln(T_{t-1} Y_{t-1}) + \alpha_2 \ln((1 - T_{t-1}) Y_{t-1})$$

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Data Description

Table 1. Economic growth and tax burden

Year	GDP, bln MNT	Economic growth	Tax burden	*Year*	GDP, bln MNT	Economic growth	Tax burden
1991	4,774.3	−8.7	0.319	2005	7,128.3	7.3	0.275
1992	4,330.3	−9.3	0.228	2006	7,738.3	8.6	0.338
1993	4,191.7	−3.2	0.253	2007	8,531.3	10.2	0.379
1994	4,279.7	2.1	0.225	2008	9,290.6	8.9	0.331
1995	4,553.6	6.4	0.222	2009	9,172.7	−1.3	0.303
1996	4,653.8	2.2	0.221	2010	9,756.6	6.4	0.320
1997	4,835.3	3.9	0.239	2011	11,443.6	17.5	0.340
1998	4,994.9	3.3	0.254	2012	12,853.4	12.5	0.297
1999	5,149.7	3.1	0.247	2013	14,350.7	11.6	0.312
2000	5,206.4	1.1	0.280	2014	15,482.3	8.1	0.284
2001	5,362.6	3.0	0.316	2015	15,847.2	2.5	0.258
2002	5,614.6	4.7	0.308	2016	16,047.8	1.4	0.244
2003	6,007.9	7.0	0.303	2017	16,873.4	5.2	0.284
2004	6,646.2	10.6	0.302	2018	18,059.5	7.0	0.313

Source: National Statistics Office

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Numerical Results

For econometric analysis we use the relative data which shows relationship between economic growth and tax burden of Mongolia for period 1991-2018 (Table 1). Econometric analysis of Schully model [7] which employs the production function with constant returns to scale gives the following result:

$$\ln\left(\frac{1+\theta}{1-T}\right) = 0.764 + 0.413\ln\left(\frac{T}{1-T}\right), \qquad R^2 = 0.73, DW = 0.61 \tag{7.1}$$

Tax rate $\alpha_1 = 0.413$ is too high for Mongolia economy so this estimation is not acceptable in practice. But estimation of parameters of the model by econometric analysis without constraints on parameters leads to the equation:

$$\ln(Y_t) = 0.5 + 0.356\ln(T_{t-1}Y_{t-1}) + 0.668\ln((1 - T_{t-1})Y_{t-1}),$$
$$R^2 = 0.99, DW = 0.97 \tag{7.2}$$

If we compare the value of R-squared with the previous one, this was increased by 0.26. However, Durbin-Watson's test is low so there is supposed to be a long-term equilibrium between independent and dependent variables. Johansen Cointegration test allows determining the number of cointegrated equations between the integrated terms with the same order.

Table 2. Johansen cointegration test

H_0	H_1	*Trace*	5%	H_0	H_1	*Max*	5%
$r = 0$	$r \geq 1$	22.6	29.79	$r = 0$	$r = 1$	15.89	21.13
$r \leq 1$	$r \geq 2$	6.71	15.49	$r = 1$	$r = 2$	6.67	14.26
$r \leq 2$	$r \geq 3$	0.03	3.87	$r = 2$	$r = 3$	0.032	3.84

The empirical results show that the null hypothesis ($r = 0$) or ($r \leq 1$) for Trace test and null hypothesis ($r = 0$) or ($r = 1$) for Maximum Eigen-value test which was not rejected at 5 percent. Consequently, these two cointegration tests cannot be confirmed that variables are cointegrated. Then, the optimal tax rate computed by (3.6) is

$$T^* = \frac{0.356}{0.356 + 0.668} = 0.348$$

This result is still high so it cannot be accepted. That is why we need to implement the constrained regression model. For this case, in order to estimate parameters of non-constant return production function by econometric model, we solve the following constrained convex minimization problem.

$$\min F(\alpha_0, \alpha_1, \alpha_2) = \sum_{i=1}^{m} [\ln \alpha_0 - \ln Y_{i-1} - \ln(1+\theta) +$$

$$+\alpha_1(\ln T_i + \ln Y_{i-1}) + \alpha_2(\ln(1-T_i) + \ln Y_{i-1})]^2$$

subject to constraint

$$\alpha_1 + \alpha_2 \leq 1,$$
$$\alpha_1 \geq 0, \alpha_2 \geq 0.$$

The problem was solved on Mongolian economic data from 2009 to 2018 by MATLAB. The solution was $\alpha_1 = 0.29$, $\alpha_2 = 0.70$. Then, the optimal tax rate computed by (3.6) is

$$T^* = \frac{0.29}{0.29 + 0.7} = 0.296$$

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Conclusion

We defined the optimal tax rate to provide economic growth of Mongolian economy based on Scully's model. We improve and modify the model by providing the concavity of the production function used in it and also reducing the parameter estimation problem to constrained optimization problem.

We find an optimal tax rate as 29.6 percent of the GDP of Mongolia which is greater than average tax rate 28.6 by on percent. It means that at this point of tax rate economic growth will reach the maximum and policymakers should take this value into account in their decision making.

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Thank you for your attention.