ECP 2ème année
Semestre 4, Electif 9, Année 2011
Mr DUMAS et Mr BOULMEZAOUD

NUMERICAL OPTIMIZATION AND APPLICATIONS, "rattrapage"

2011, May 12th, 3 hours

Documents and laptops are allowed, only at the second half of the exam.

All Scilab programs must be sent at the adress `laurent.dumas@uvsq.fr` *at the end of the exam with a file name of the type* `ECP2011-firstname-name.sci` *and with the subject ECP2011. The problem is in english but candidates can give a copy in french if they prefer.*

A. Theoretical part

Exercice 1 – Use the simplex method to solve the linear program

$$\begin{aligned} \max 2x + y, \\ x \geq 0, \\ y \geq 0, \\ x + 2y \leq 6, \\ x + y \leq 4, \\ x \leq 3. \end{aligned}$$

Exercice 2 – Let $\alpha > 0$ be a fixed real number. Consider the function f defined on $\mathbb{R}^2$ by

$$f(x, y) = x + \alpha y, \quad \text{for all } (x, y) \in \mathbb{R}^2.$$

Consider the set

$$D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + \alpha y^2 = 1\}.$$

The objective here is to solve the problem

$$\min_{(x,y)\in D} f(x, y). \tag{1}$$

1. Is D convex? Is-it closed?
2. Show that D is bounded.
3. Deduce that problem (1) admits at least one solution (it is not required to find it).
4. Compute $\nabla f(x, y)$, the gradient of f at a typical point $(x, y) \in \mathbb{R}^2$.
5. Solve (1).

B. Computational part

Starting from an existing code, the objective is to make some modifications and/or improvements of it. The initial script is given in annex and can be downloaded at the following adress:

`http://www.math.uvsq.fr/~dumas/GA-binary2011.sci`.

This code has been written during a computer session and has been called 'the historical GA'.

1. Run the 'historical GA' and explain the obtained result. Why is it a good optimization algorithm?
2. Implement a new selection method called '2-tournament'. The principle is the following: in order to create the mating pool, select randomly 2 elements of the population and put in the mating pool the best element between them. Repeat $Npop$ times this operation in order to create a mating pool of $Npop$ elements.
3. Implement the elitism principle in the following way: at the end of each generation, verify if the cost function for the best element has decreased. If not, take randomly an element of the current population and replace it by the best element of the population of the previous generation.
4. Compare the performance of the new script with the initial one on a simple test case.