ECP Year 2Semestre 3, Electif 9,
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NUMERICAL OPTIMIZATION ANDAPPLICATIONS
Practical/Tutorial Session

29/02/12

Q1 Consider the following function of two variables:

$$F(\mathbf{x}) = (x_1 + x_2)^4 - 12x_1x_2 + x_1 + x_2 + 1$$

i. Verify that the function has three stationary points at

$$\mathbf{x}^1 = \begin{bmatrix} -0.6504 \\ -0.6504 \end{bmatrix} \qquad \mathbf{x}^2 = \begin{bmatrix} 0.085 \\ 0.085 \end{bmatrix} \qquad \mathbf{x}^3 = \begin{bmatrix} 0.5655 \\ 0.5655 \end{bmatrix}$$

ii. Test the stationary points to find any minima, maxima or saddle points.
iii. Find the second-order Taylor series approximations for the function at each of the stationary points.
iv. Plot the function and the approximations using MATLAB.

Q2 We want to find the minimum of the following function:

$$F(X) = \frac{1}{2} X^T \begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix} X + \begin{bmatrix} -1 & -1 \end{bmatrix} X$$

i. Sketch a contour plot of this function.
ii. Sketch the trajectory of the steepest descent algorithm on the contour plot of part (i), if the initial guess is $X_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Assume a very small learning rate is used.
iii. Perform two iterations of steepest descent with learning rate $\alpha = 0.1$.
iv. What is the maximum stable learning rate?
v. What is the maximum stable learning rate for the initial guess given in part (ii)?
vi. Write a MATLAB M-file to implement the steepest descent algorithm for this problem, and use it to check your answers to parts (i) through (v).

Q3 Recall the function presented in **Q1**. Write MATLAB M-files to implement the Steepest Descent algorithm and Newton's method for that function. Test the performance of the algorithms for various initial guesses.