

Q1 Consider the following function of two variables:

$$F(\mathbf{x}) = (x_1 + x_2)^4 - 12x_1x_2 + x_1 + x_2 + 1$$

i. Verify that the function has three stationary points at

$$\mathbf{x}^1 = \begin{bmatrix} -0.6504 \\ -0.6504 \end{bmatrix} \quad \mathbf{x}^2 = \begin{bmatrix} 0.085 \\ 0.085 \end{bmatrix} \quad \mathbf{x}^3 = \begin{bmatrix} 0.5655 \\ 0.5655 \end{bmatrix}$$

- ii. Test the stationary points to find any minima, maxima or saddle points.
- iii. Find the second-order Taylor series approximations for the function at each of the stationary points.
- iv. Plot the function and the approximations using MATLAB/SCILAB.

Q2 We want to find the minimum of the following function:

$$F(X) = \frac{1}{2} X^T \begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix} X + \begin{bmatrix} -1 & -1 \end{bmatrix} X$$

- i. Sketch a contour plot of this function.
- ii. Sketch the trajectory of the steepest descent algorithm on the contour plot of part (i), if the initial guess is $X_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Assume a very small learning rate is used.
- iii. Perform two iterations of steepest descent with learning rate $\alpha = 0.1$.
- iv. What is the maximum stable learning rate?
- v. What is the maximum stable learning rate for the initial guess given in part (ii)?
- vi. Write a MATLAB/SCILAB program to implement the steepest descent algorithm for this problem, and use it to check your answers to parts (i) through (v).

Q3 Recall the function presented in **Q1**. Write a MATLAB/SCILAB program to implement the Steepest Descent algorithm and Newton's method for that function. Test the performance of the algorithms for various initial guesses.