Course on Optimization
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Exercise session 1: first and second order methods

Question 1

Graph few contour lines of the function given by:

$$J(x,y) = 2(x-1)^2 + y^2 + 1$$

then at a given point on the contour lines, draw the gradient.

Question 2

Let $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ be differentiable and convex. Let x and y be elements of $\mathbb{R}^n$, such as $f(y) < f(x)$. Show that $y - x$ is a descent direction of f at x. It is always the case when f is not convex?

Question 3

The following function is to be minimized on $\mathbb{R}^3$:

$$J(x,y,z) = x^2 + 2y^2 + z^2 - x + y - z.$$

a Give the exact solution of the problem considered, explaining your answer clearly.

b The gradient descent method, with a backtracking strategy using Armijo condition is used in order to find the numerical solution using the initial approximation $X_0 = (0,1,1)$.

(i) What is the descent direction d at the first iteration?

(ii) For the function J, defined above, calculate explicitly and represent graphically in this case the function $\alpha \mapsto J(X_0 + \alpha d)$. Find the corresponding values of α satisfying the Armijo condition for $\beta = 0.1$, that is:

$$J(X_0 + \alpha d) \leq J(X_0) + \beta\alpha\langle d, \nabla J(X_0)\rangle$$

c Find X_1 satisfying the Armijo condition using the backtracking algorithm for $\alpha_{init} = 1$ and $\tau = 0.1$. Recall that the backtracking method consists in finding the step size α by trying succesively α_{init}, then $\tau\alpha_{init}$, $\tau^2\alpha_{init}$, ..., until it satisfies the Armijo condition.

d Compute the sequence X_k for $k \geq 2$ by using a Python script. Does it converge to the exact minimum?

Question 4

It is required to minimize the following function:

$$J(x, y, z) = x^4 + 2y^4 + z^4 - 2x + y - z$$

on $\mathbb{R}^3$.

a Give the exact solution of the above minimization problem.

b Newton's method is used in order to obtain a numerical solution using the initial approximation $X_0 = (1, 1, 1)$. Find X_1.

d Compute the sequence X_k for $k \geq 2$ by using a Python script. Does it converge to the exact minimum?