Course on Optimization University of Mauritius, January 2023
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Exercise session 2: derivative free optimization, direct methods

Question 1 *On the Nelder Mead algorithm*

Consider the Nelder Mead algorithm for the sphere function in $\mathbb{R}^2$:

$$f(x,y) = x^2 + y^2$$

and the initial simplex made of $A = (4,5)$, $B = (5,3)$ and $C = (5,6)$.

Compute the best obtained value by this algorithm after one iteration.

Question 2 *On the Nelder Mead algorithm*

1. Recall briefly the main principles of the Nelder Mead algorithm. A 2D illustration of the possible steps can be used.

2. Prove that no shrinkage steps are performed when the Nelder Mead algorithm is applied to a strictly convex function. We recall that $f : \mathbb{R}^n \to \mathbb{R}$ is strictly convex if and only if:

   $$\forall (x,y) \in \mathbb{R}^n \times \mathbb{R}^n,\ \forall \lambda \in]0,1[,\ f(\lambda x + (1-\lambda)y) < \lambda f(x) + (1-\lambda) f(y) \text{ if } x \neq y$$

Question 3 *On the Nelder Mead algorithm*

Consider the Nelder Mead algorithm with its nominal parameters. Denote $S_k = \{y_k^0, ..., y_k^n\}$ the simplex obtained at iteration k with the corresponding values by the function f: $f_k^0 \leq f_k^1 \leq ... \leq f_k^n$.

Assume that the function f is bounded from below

1. Prove that the sequence $(f_k^0)_{k\in\mathbb{N}}$ is convergent.

2. If only a finite number of shrink steps occurs, prove that all the sequences $(f_k^i)_{k\in\mathbb{N}}$ are convergent ($0 \leq i \leq n$).

3. Define the volume of the simplex:

$$V(S_k) = \frac{1}{n!}\det[y_k^0 - y_k^n, ...y_k^{n-1} - y_k^n]$$

Check that for $n = 2$ the definition of $V(S_k)$ corresponds to the area of the triangle (S_k).

4. Give the value of $V(S_{k+1})$ in terms of $V(S_k)$ when an expansion occurs (for sake of simplicity, we can assume that $y_k^n = 0$).

5. Give the value of $V(S_{k+1})$ in terms of $V(S_k)$ when a shrink step occurs.