Université de Versailles — Année 2019-2020

http://dumas.perso.math.cnrs.fr/V04.html — Master AMS, module V04

DERVATIVE FREE OPTIMIZATION
FINAL EXAM, DETERMINISTIC PART

Exercice 1 *On the Nelder Mead algorithm*

Consider the Nelder Mead algorithm with its nominal parameters. Denote $S_k = \{y_k^0, ..., y_k^n\}$ the simplex obtained at iteration k with the corresponding values by the function f : $f_k^0 \leq f_k^1 \leq ... \leq f_k^n$.

Assume that the function f is bounded from below

1. Prove that the sequence $(f_k^0)_{k\in\mathbb{N}}$ is convergent.
2. If only a finite number of shrink steps occurs, prove that all the sequences $(f_k^i)_{k\in\mathbb{N}}$ are convergent $(0 \leq i \leq n)$.
3. Define the volume of the simplex :

$$V(S_k) = \frac{1}{n!}\det[y_k^0 - y_k^n, ...y_k^{n-1} - y_k^n]$$

   Check that for $n = 2$ the definition of $V(S_k)$ corresponds to the area of the triangle (S_k).
4. Give the value of $V(S_{k+1})$ in terms of $V(S_k)$ when an expansion occurs (for sake of simplicity, we can assume that $y_k^n = 0$).
5. Give the value of $V(S_{k+1})$ in terms of $V(S_k)$ when a shrink step occurs.

Exercice 2 *On a DFO trust region method*

Consider a DFO trust region method : denote $f : \mathbb{R}^n \to \mathbb{R}$ the function to minimize and m_k the quadratic model at iteration k around the current point x_k (in a ball of radius Δ_k). Assume that the hessian of m_k at x_k, H_k, has at least one negative eigenvalue and let τ_k its minimal eigenvalue. Denote g_k the gradient of m_k at x_k.

1. Recall the general principles of a DFO trust region method
2. Give the expression of $s \mapsto m_k(x_k + s)$ with respect to g_k and H_k.
3. Prove that there exists a vector $s_k \in \mathbb{R}^n$ such that

$$\begin{cases} < s_k, g_k > \leq 0 \\ ||s_k|| = \Delta_k \\ < s_k, H_k s_k > = \tau_k \Delta_k^2 \end{cases}$$

4. Give a positive lower bound of $m_k(x_k) - m_k(x_k + s_k)$ in terms of τ_k and Δ_k.

Exercice 3 *On the DIRECT method*

Consider the 1D function $f(x) = x^4 - 2x$ to be minimized on the interval $[-1, 3]$.

Make three iterations of the DIRECT method on this case.

We recall below its general principle (issued from Jones et al, JOGO, 1993) :

Univariate DIRECT Algorithm.

Step 1. Set $m = 1$, $[a_1, b_1] = [\ell, u]$, $c_1 = (a_1 + b_1)/2$, and evaluate $f(c_1)$. Set $f_{\min} = f(c_1)$. Let $t = 0$ (iteration counter).

Step 2. Identify the set S of potentially optimal intervals.

Step 3. Select any interval $j \in S$.

Step 4. Let $\delta = (b_j - a_j)/3$, and set $c_{m+1} = c_j - \delta$ and $c_{m+2} = c_j + \delta$. Evaluate $f(c_{m+1})$ and $f(c_{m+2})$ and update $f_{\min}$.

Step 5. In the partition, add the left and right subintervals

$[a_{m+1}, b_{m+1}] = [a_j, a_j + \delta]$, center point c_{m+1},

$[a_{m+2}, b_{m+2}] = [a_j + 2\delta, b_j]$, center point c_{m+2}.

Then modify interval j to be the center subinterval by setting

$[a_j, b_j] = [a_j + \delta, a_j + 2\delta]$.

Finally, set $m = m + 2$.

Step 6. Set $S = S - \{j\}$. If $S \neq \emptyset$, go to Step 3.

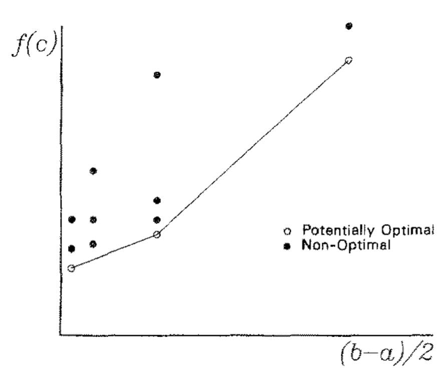

Fig. 6. Set of potentially optimal intervals.