

## Inverse problems for blood flow simulation

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### 1. Abstract

A 1D model for the simulation of blood flow in arteries is studied here. Derived from the general 3D equations, it computes the section area of the artery and the volumic flux of the flow for each longitudinal position and time. The present article deals with the identification of the main parameter of this model linked to the artery compliance. Even in the case of an artery experiencing a loss of compliance in some portion, the inverse problem is successfully solved by using a reduced number of experimental or numerical measurements.

**2. Keywords :** Optimization, blood flow, 1D models.

### 3. Introduction

In order to reduce the cost of complex 3D fluid-structure computations of blood flow in arteries, a one dimensional model have been developed based on the averaging of the general three dimensional equations [1, 3]. Under a certain number of hypotheses for the artery flow and geometry, it computes the section area  $A(t, z)$  and the volumic flux  $Q(t, z)$  at any longitudinal position  $z$  and time  $t$ . Such model is interesting for three main reasons : first, it drastically reduces the computational time of the 3D model, secondly, its two unknowns are quantities that can be experimentally obtained by non invasive techniques and finally, it allows to recover all the other hemodynamic variables like blood pressure that are more difficult to measure.

The general objective of this work is to show that the parameters of this 1D model can be fitted either with the results of 3D computations or with experimental values measured at a finite number of positions  $z$  and times  $t$ . In particular, it includes the case of an artery with a large compliance step which occurs when an artery is partially obturated by atherosclerotic plaques. In this case, it can be important to locate precisely the position of the plaque before putting a permanent metallic implant used to prop open arteries and called stent [2]. Even after this operation, note that the artery will still experience a compliance step at the stent location.

The main parameter of the 1D model that has to be fixed is a function of  $z$ , called  $\beta$ , directly linked to the compliance value of the artery. In order to do so, an inverse problem is solved by means of Genetic Algorithms. In section 4, the direct computation of the 1D model is presented in two particular cases : a healthy artery with a fixed compliance and a disease one with a large loss of compliance between two points. In section 5, the corresponding inverse problem is solved by fitting the 1D model with synthetic data, issued either from a 3D or a 1D computation. The problem of the amount of information needed to recover the correct shape of  $\beta$  is in particular discussed.

### 4. The 1D model : presentation and resolution of the direct problem

#### 4.1 From 3D to 1D

The blood velocity and pressure in an artery satisfy the following Navier Stokes equations in a 3D domain similar to the one depicted in Figure 1 :

$$\begin{cases} \frac{\partial u}{\partial t} + (u \cdot \nabla)u + \frac{1}{\rho} \nabla p + \operatorname{div} \left( \nu \frac{\nabla u + (\nabla u)^T}{2} \right) = 0 \\ \operatorname{div}(u) = 0 \end{cases} \quad (1)$$

where  $\rho$  and  $\nu$  respectively denote the density and the kinematic viscosity of the blood. Because of the compliance of the arterial wall, these equations must be coupled with an appropriate model of the

vessel wall dynamics. Moreover, this compliance can vary along the artery, for instance because of the formation of atherosclerotic plaques or the presence of a stent (see Figure 1).

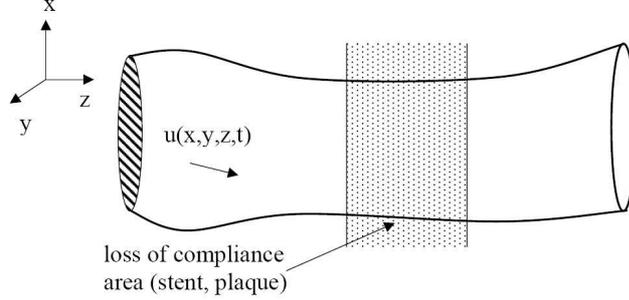


FIG. 1 – Blood flow in an artery : the 3D model

In order to reduce the computational time needed to solve these equations, a 1D model has been proposed by assuming that the artery has a cylindrical geometry with a circular cross section (see Figure 2).

After averaging the 3D Navier Stokes equations over each cross section, a system with two unknowns,  $A(t, z)$  the section area at time  $t$  and position  $z \in [0, L]$ , and  $Q(t, z)$  the volumic flux, is derived (see [1] for more details) :

$$\begin{cases} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial z} = 0 \\ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial z} \left( \frac{Q^2}{A} \right) + \frac{A}{\rho} \frac{\partial p}{\partial z} + K_r \frac{Q}{A} = 0 \end{cases} \quad (2)$$

In this system,  $p$  denotes the average pressure and is determined by the following law :

$$p(t, z) = \beta(\sqrt{A} - \sqrt{A_0}) \quad (3)$$

where  $A_0$  represents the section area of the artery at rest.

The coefficients  $K_r$  and  $\beta$  appearing in the model are respectively linked to the blood viscosity and the elastic properties of the artery. In view of the possible applications,  $\beta$  will thus take the form of a function of  $z$ . A formal derivation from the 3D to the 1D model lead to the following expression for the parameter  $\beta$  :

$$\beta(z) = \frac{4\sqrt{\pi}h_0E(z)}{3A_0} \quad (4)$$

where  $h_0$  and  $E(z)$  respectively denote the thickness and the Young modulus of the vessel wall.

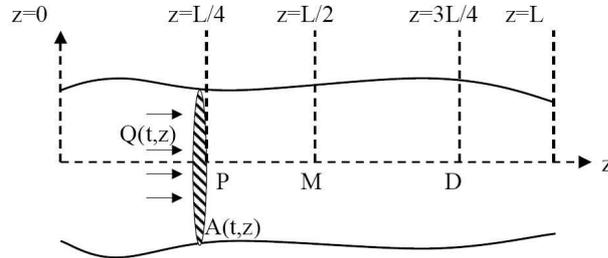


FIG. 2 – Blood flow in an artery : the 1D model

The 1D model (2)-(3) can be rewritten in a conservative form :

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial z} = B(U) \quad (5)$$

where  $U = \begin{pmatrix} A \\ Q \end{pmatrix}$  and

$$F(U) = \begin{pmatrix} Q \\ \frac{Q^2}{A} + \frac{\beta}{3\rho} A^{\frac{3}{2}} \end{pmatrix}, \quad B(U) = \begin{pmatrix} 0 \\ -K_r \frac{Q}{A} + \frac{A}{\rho} \frac{d\beta}{dz} (\sqrt{A_0} - \frac{2}{3} \sqrt{A}) \end{pmatrix}$$

It can be observed that this system is hyperbolic, as the Jacobian

$$H = \frac{\partial F}{\partial U} = \begin{pmatrix} 0 & 1 \\ \frac{\beta\sqrt{A}}{2\rho} & -(\frac{Q}{A})^2 + 2\frac{Q}{A} \end{pmatrix}$$

has always two real and distinct eigenvalues for all the allowable values of  $U$ . The system (5) is then completed by two appropriate boundary conditions, one at each end. A pressure profile is in particular imposed at the entrance (see [1] for more details).

#### 4.2 Numerical discretization

The equations of the 1D model are discretized in their conservative form (5) by using a second order Taylor Galerkin scheme. Denote  $\Delta t$  the chosen time step, then the vector of unknowns  $U^n$  at time  $t_n = n\Delta t$  satisfies the following time-marching scheme :

$$U^{n+1} = U^n - \Delta t \frac{\partial}{\partial z} (F^n + \frac{\Delta t}{2} \frac{\partial F^n}{\partial U} B^n) - \frac{\Delta t^2}{2} (\frac{\partial B^n}{\partial U} \frac{\partial F^n}{\partial z} - \frac{\partial}{\partial z} (\frac{\partial F^n}{\partial U} \frac{\partial F^n}{\partial z})) + \Delta t (B^n + \frac{\Delta t}{2} \frac{\partial B^n}{\partial U} B^n)$$

The spatial discretization is then done by using linear finite elements on the subdivision  $(z_i)_{i \in \{0, \dots, N\}}$  of  $[0, L]$  (see [1] for more details).

#### 4.3 Numerical tests

Two numerical tests have been performed in order to see the influence of a compliance loss on the transmission of a pressure impulse. In both cases, the same pulsatile pressure, displayed on Figure 3, is imposed at the entrance of an artery of length  $L = 4\text{cm}$ .

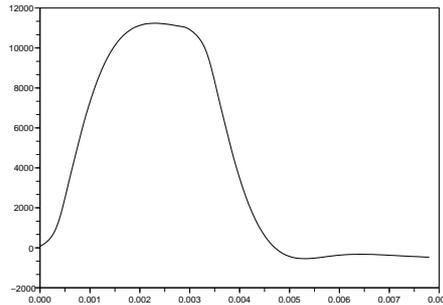


FIG. 3 – Blood pressure at entrance (in dynes/cm<sup>2</sup>)

For both cases, the section area at rest  $A_0$  is chosen constant and equal to 0.780362 cm<sup>2</sup> whereas  $K_r = 0.75$ . On the contrary, the coefficient  $\beta$  is different between the two test cases (see below).

The initial conditions for  $A$  and  $Q$  are chosen constant in space, respectively  $A(0, z) = A_0$  and  $Q(0, z) = 0$ .

##### *Test cas 1 : artery with a constant compliance*

In this first test case, the compliance is chosen constant and taken equal to its formal expression (4)

with  $E_0 = 4.10^6$  dynes/cm<sup>2</sup> and  $h_0 = 0.1$  cm, that is :

$$\beta_{formal} = 1211372 \text{ dynes/cm}^3$$

A uniform spatial discretization of  $[0, L]$  is chosen with  $N = 128$  and the time step is taken equal to  $10^{-5}$ .

The results are depicted on Figure 4. The two unknowns  $A$  and  $Q$ , as well as the pressure issued from them by the relation (3), are displayed at three different points inside the artery, respectively located at  $z = \frac{L}{4}, \frac{L}{2}$  and  $\frac{3L}{4}$  and called points  $P, M$  and  $D$  (see Figure 2).

In this case, it can be observed that the pressure impulse travels along the artery at a constant speed and with minor changes during its propagation.

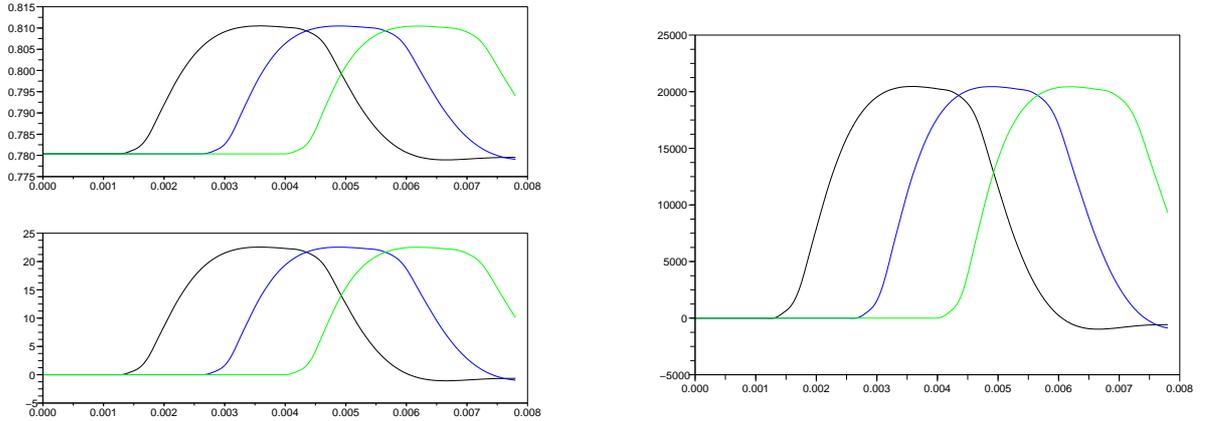


FIG. 4 – Section area  $A$  (top left), volumic flux  $Q$  (bottom left) and pressure (right) at points  $P, M$  and  $D$  for the constant compliance case.

*Test case 2 : artery with a variable compliance*

The second test case studies the propagation of the same impulse pressure in a case where the compliance of the artery is supposed to be variable with  $z$ .

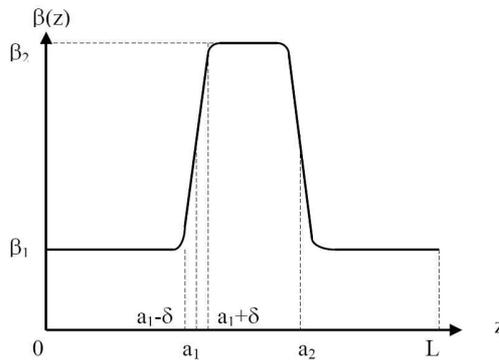


FIG. 5 – Function  $z \rightarrow \beta(z)$ , variable compliance case

More precisely, the value of  $\beta$  is taken 10 times higher in a portion located between  $a_1 = 1.5$  cm and  $a_2 = 2.5$  cm. Such abrupt change can be linked for instance by the presence of a stent or of atherosclerotic plaques at this location inducing a severe loss of compliance. Numerically, the function  $\beta \rightarrow \beta(z)$  has to be at least  $C^2$  because of the presence of the term  $\frac{d\beta}{dz}$  in  $B(U)$ . A smoothing procedure with a small

transition parameter  $\delta$  is thus done leading to the shape depicted in Figure 5 with

$$(\beta_1, \beta_2) = (\beta_{formal}, 10\beta_{formal}) \quad \text{and} \quad (a_1, a_2) = (1.5, 2.5)$$

Because of the large value of  $\frac{d\beta}{dz}$  near the points  $z = a_1$  and  $z = a_2$ , the spatial mesh has to be refined locally and the time step is now equal to  $3.10^{-6}$ .

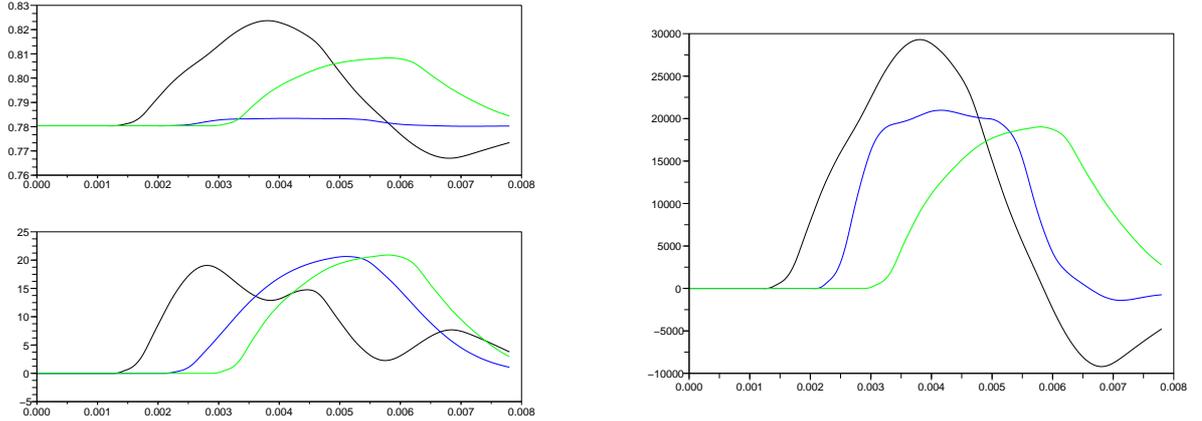


FIG. 6 – Artery section (top left), blood flow rate (bottom left) and pressure (right) at points  $P$ ,  $M$ ,  $D$  for the variable compliance case.

The results depicted on Figure 6 show that the pressure wave is partially reflected at  $z = a_1$ , thus giving a more complex pressure profile downstream and also a decrease of the next pressure peaks. Note also that the propagation speed increases inside the disease portion while its section area remains almost constant.

## 5 The 1D model : resolution of the inverse problem

### 5.1 Objectives and resolution method

The aim of this section is to fit the 1D model, more precisely the value of the parameter  $\beta$  (variable or not), in order to be able to recover, either a 3D computation or an experimental result. Another problem that is studied here is to find the amount of information needed on the 1D model that would allow to recover all its parameters.

Both objectives are in the domain of parameters identification and can thus be view as inverse problems. To find the global minima of such complex cost functions, Evolutionary Algorithms like Genetic Algorithms have shown their efficiency for more than a decade. Inspired from the Darwinian theory of evolution of species [4], they have been applied in numerous applicative domains including the biomedical field, cite for instance the improvement of the aerodynamics of car shapes [5] or of various medical devices like pacemakers [5] or stents [6].

### 5.2 The inverse problem with a constant compliance

The first inverse problem that is studied here concerns the case of an artery with a constant compliance. The objective is to find the optimal constant  $\beta$  that would allow the 1D model to fit as best as possible the corresponding 3D computation. In particular, it will corroborate or deny the formal expression (4) as the best choice for  $\beta$ .

To this aim, a preliminary 3D computation with a fluid structure model has first been done on an artery of the same type as the one used in the previous section and with the same pressure impulse. The 3D Navier Stokes equations (1) with a linear Venant Kirchoff model for the structure have been solved with the C++ library *LifeV* (see [3] for more details). The results of this computation at a particular time are given on Figure 7.



FIG. 7 – Solution of the 3D model at  $t = 50.10^{-4}$  (left : fluid velocity, right : wall displacement)

To find the optimal value of  $\beta$  for the associated 1D model, an error-type cost function is defined. It uses the computed value with the 3D model of the section area  $A_{3D}$  and the volumic flux  $Q_{3D}$  for the three points  $P$ ,  $M$  and  $D$  previously defined (see Figure 2) and at the times  $t_i = i10^{-4}$  ( $0 \leq i \leq 78$ ). More precisely :

$$J_1(\beta) = \sum_{pts \in \{P, M, D\}} \sum_{i=0}^{78} \left( |A(t_i, pts) - A_{3D}(t_i, pts)|^2 + \frac{1}{10} |Q(t_i, pts) - Q_{3D}(t_i, pts)|^2 \right)$$

In [3], the same problem had been tackled by using a deterministic optimization method and by computing the gradient of  $J_1$  with an adjoint formulation. It is solved here by using a real valued Genetic Algorithm with 100 evaluations of the direct problem (5 individuals and 20 generations). As the shape of the function  $\beta \rightarrow J_1(\beta)$  exhibits a convex behavior, both methods succeed to locate the global minimum of  $J_1$ , namely

$$\beta_{opt} = 0.5490284 * \beta_{formal} = 665077$$

It is interesting to notice that the optimal value of  $\beta$  which allows the 1D model to fit to the corresponding 3D model is far below the value issued from a formal derivation.

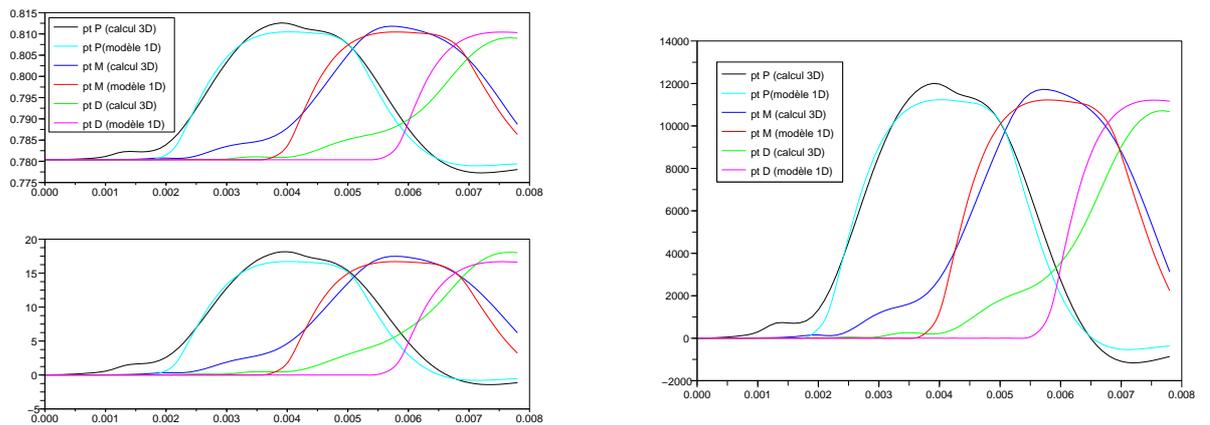


FIG. 8 – Section area (top left), volumic flux (bottom left) and pressure (right) at points  $P$ ,  $M$  and  $D$  for the 3D and the optimal 1D case, artery with constant compliance.

A comparison between the values of section area, volumic flux and pressure issued from the 3D model and the 1D model with  $\beta = \beta_{opt}$  at points  $P$ ,  $M$  and  $D$  is visible on Figure 8. It shows in particular

that the 1D model can fit relatively well with the complex 3D model at a very lower computational cost.

### 5.3 The inverse problem with a variable compliance

The second inverse problem that is studied concerns the case of arteries having a compliance step that can be due to different reasons (stent, plaques,...). The objective now is to determine the characteristics of this compliance step by looking at the values of the section area  $A$  and/or the volumic flux  $Q$  at different positions. In the present case, the synthetic data are issued from a direct computation of the 1D model already presented in section 4.3 (Test case 2).

In order to determine the amount of informations needed to recover the correct parameters of the model, two test function have been tested. Both have four parameters as unknowns : the position of the compliance loss,  $a_1$  and  $a_2$ , and the respective values,  $\beta_1$  and  $\beta_2$  of the function  $z \rightarrow \beta(z)$  in both zones,(see Figure 5). The two cost functions write respectively as :

$$J_2(a_1, a_2, \beta_1, \beta_2) = \sum_{pts \in \{P, M, D\}} \sum_{i=0}^{78} |A(t_i, pts) - A_{target}(t_i, pts)|^2 + \frac{1}{10} |Q(t_i, pts) - Q_{target}(t_i, pts)|^2$$

and

$$J_3(a_1, a_2, \beta_1, \beta_2) = \sum_{i=0}^{78} |A(t_i, D) - A_{target}(t_i, D)|^2$$

which means that the optimization is done with much less informations in the case of  $J_3$ , namely the section area values down the compliance step.

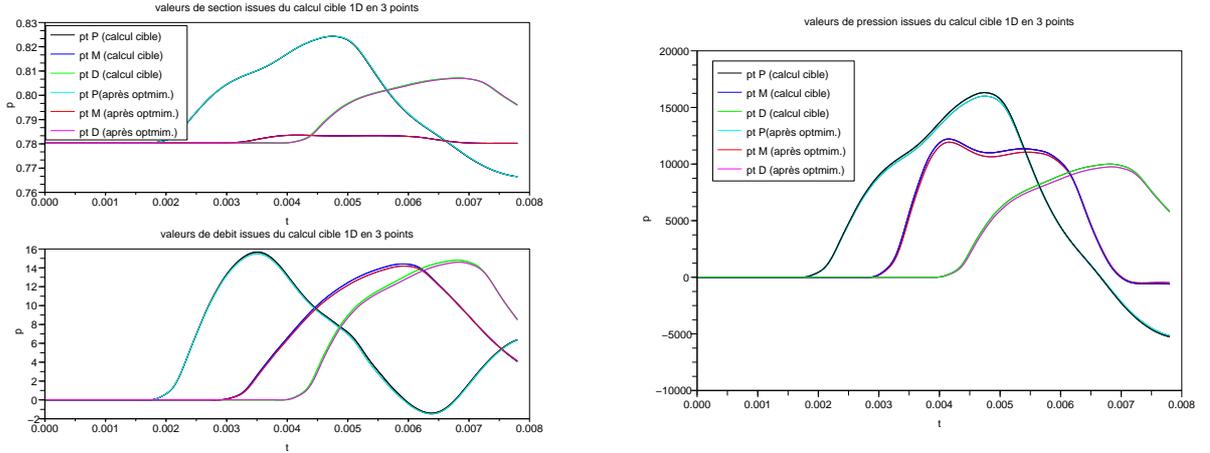


FIG. 9 – Section area, volumic flux and pressure at points  $P$ ,  $M$  and  $D$  for the synthetic data and the optimal 1D case, artery with variable compliance, cost function  $J_2$ .

Both optimization process have been done by using a real-valued Genetic Algorithm with less than 400 evaluations. In both cases the four parameters have been chosen in a very large domain. In particular,  $a_1$  and  $a_2$  are allowed to take all the values between 0 and 4 with  $a_1 < a_2$ .

In the case of the cost function  $J_2$ , the four parameters characterizing the function  $z \rightarrow \beta(z)$  have been recovered with an excellent accuracy. It explains the perfect matching visible on Figure 9 of the values of the optimal 1D model with the synthetic data.

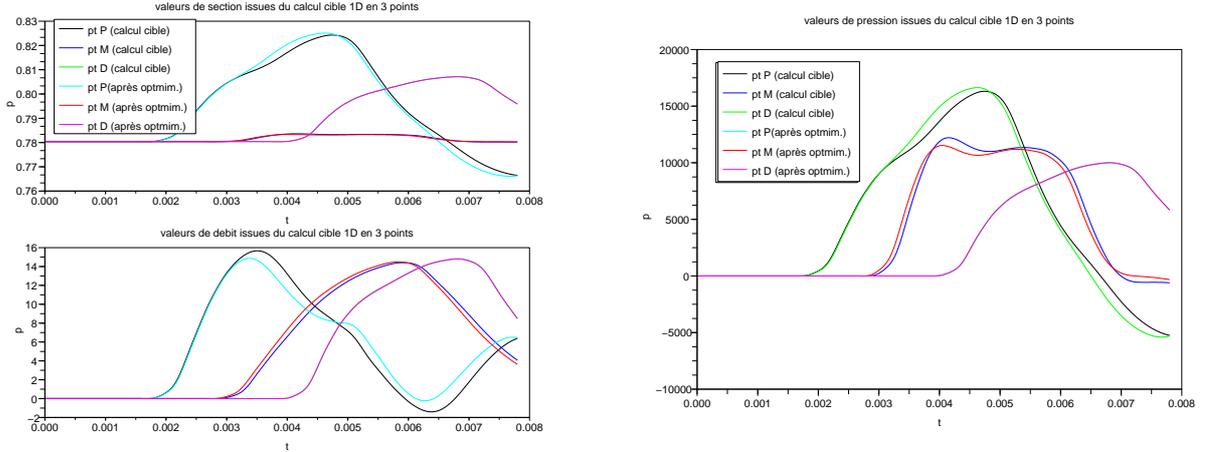


FIG. 10 – Section area, volumic flux and pressure at points  $P$ ,  $M$  and  $D$  for the synthetic data and the optimal 1D case, artery with variable compliance, cost function  $J_3$ .

In the case of the cost function  $J_3$ , the situation is slightly different as the optimization process is only able to locate the position of the compliance step (that is  $a_1$  and  $a_2$ ) but not the exact amount of it. More precisely, the obtained value of  $\beta_1$  matches the one used in the synthetic data but the value of  $\beta_2$  is underpredicted by 50%. It can be observed on Figure 10 that the profile of the section area at point  $D$  is similar for both models while there exists some significant differences at point  $P$ , that is upstream the compliance step.

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## 6. Conclusions

The inverse problem corresponding to the identification of the main parameter of a 1D model for blood flow has been successfully solved. It has shown in particular that the optimal parameters may be different from those expected from a formal averaging. Moreover, in the case of an artery with a loss of compliance in some portion, the knowledge of only one area section profile downstream is enough to locate the exact position of this disease portion. In the future, such model may thus be able to help practioners for an efficient quick diagnosis of arteries pathologies such as stenosis.

## 7. References

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