EVOLUTIONARY METHODS FOR DESIGN, OPTIMIZATION AND CONTROL
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APPLICATION OF GENETIC ALGORITHMS TO CURE HEARTBEAT PATHOLOGIES

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Abstract. The purpose of this paper is to show how genetic algorithms can improve the cure of heartbeat pathologies by optimizing the positionning of the electrodes of a pacemaker. A very first simulation on a simplified disease heart is presented and show how electrocardiograms are sensitive to the electrodes positioning.

Key words: Genetic algorithms, electrical actvity of the heart, electrocardiograms

1 Introduction

The heart behaves like a pump where the contraction is induced by an electrical impulse moving across it. In the ventricles, the propagation of the electrical signal is led by the so–called bundle of His causing a wavefront which propagates by a cell–to–cell activation. In each cell, a depolarization phase occurs corresponding to the inflow of sodium ions (causing the electrical activation) followed by a plateau phase, and then by a repolarization phase corresponding to the outflow of potassium ions. The electrical conduction of heart may be defective causing the heartbeat to be too fast, too slow or irregular. Such pathologies can be treated on using an artificial pacemaker, a small device containing a battery and electrode(s) transmitting an electrical impulse.

Our aim in this paper is to determine the optimal positioning of electrodes of a pacemaker on a disease heart. This can be interpreted as an inverse type optimization problem which can be solved with optimization tools already used in other medical applications, namely evolutionary methods. In this paper we investigate two different ways to solve this problem. The first way is to determine the optimal positionning of an electrode on a disease heart by minimizing the delay in the depolarization phase. The second way is to determine this optimal positionning by using the electrocardiogram (ECG).

The paper is organized as follows. In section 2 we give a brief description of

the modelisation of the electrical activity of the heart, in section 3 we present the optimization principles. Finally, section 4 is devoted to the description of the test case and the numerical results performed on it.

2 Modelisation of the electrical activity of the heart

At the microscopic level, the cardiac muscle, denoted by Ω_H, is made of two distinct and intricate media: the intra and extra-cellular media, respectively called Ω_{Hi} and Ω_{He}, that are separated by a surface membrane Γ_H (see Figure 1).

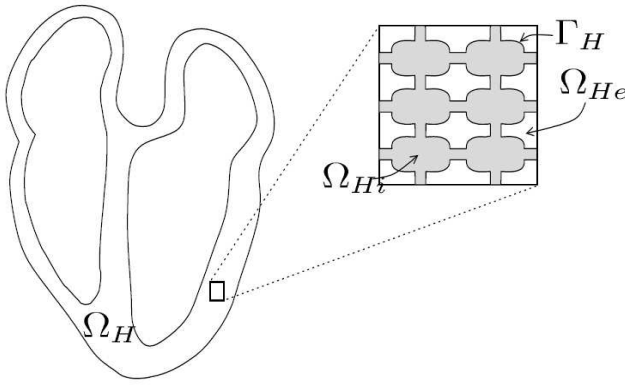

Figure 1: Simplified view of the heart at macro/microscopic level.

After a homogenization process, the corresponding electrical potentials ϕ_i and ϕ_e and the transmembrane potential

$$V_m(t,x) = \phi_i(t,x) - \phi_e(t,x) \tag{1}$$

are defined on the entire domain $x \in \Omega_H$ and satisfy the so–called bidomain model [1], on $[0,T] \times \Omega_H$:

$$A_m \left(C_m \partial_t V_m + I_{\text{ion}}\right) - \text{div}(\sigma_i \nabla V_m) = \text{div}(\sigma_i \nabla \phi_e)\,, \tag{2}$$

$$\text{div}(\sigma_i \nabla \phi_i) = -\text{div}(\sigma_e \nabla \phi_e)\,, \tag{3}$$

with the following boundary condition on the heart boundary $\partial\Omega_H$:

$$\sigma_i \nabla \phi_i \cdot n = \sigma_e \nabla \phi_e \cdot n = 0\,, \tag{4}$$

where n denotes the outward unit normal at $x \in \partial\Omega_H$.

Finally an initial condition is prescribed:

$$V_m(0,x) = V_m^0(x) \quad \text{ in } \Omega_H\,. \tag{5}$$

In this model, the parameters A_m, C_m, σ_i and σ_e respectively denote the average rate of membrane surface per volume area, the membrane capacity and the intra and extracellular conductivity tensors. Note that the presence of electrodes is modelled here by adding a local source term in (2).

The current term due to ionic exchanges, I_{ion}, is evaluated with the help of the physiological model of Mitchell and Schaeffer [2]:

$$I_{\text{ion}} = -\frac{w}{\tau_1} V_m^2 (1 - V_m) + \frac{V_m}{\tau_2}\,, \tag{6}$$

where the auxiliary variable w satisfy the following ODE:

$$\frac{dw}{dt} = g(V_m, w)\,, \quad \text{with} \quad g(V_m, w) = \begin{cases} \frac{w-1}{\tau_3} & \text{if } V_m < V_g\,, \\ \frac{w}{\tau_4} & \text{if } V_m > V_g\,, \end{cases} \tag{7}$$

and $\tau_1\,, \tau_2\,, \tau_2\,, \tau_3\,, \tau_4$ and V_g are given parameters.

The model (2)–(7) is usually used when the heart is isolated. In order to derive an ECG, this model is coupled with a model of the electrical activity of the surrounding tissues. To this end, we assume that the interface between the heart and the extra–cardiac region is divided into the endocardium Γ_{endo} and the epicardium Γ_{epi}. Then, the coupling we consider is

$$\begin{cases} R_p \sigma_e \nabla \phi_e \cdot n = R_p\, C_p \frac{\partial(\phi_T - \phi_e)}{\partial t} + (\phi_T - \phi_e) & \text{on } \Gamma_{\text{epi}} \\ \sigma_e \nabla \phi_e \cdot n = \sigma_T \nabla \phi_T \cdot n & \text{on } \Gamma_{\text{endo}}\,, \end{cases} \tag{8}$$

where ϕ_T denotes the potential in the torso domain, R_p and C_p are given parameters. We refer to [4] for more details.

3 The optimization principles

3.1 The cost functions

In this paper we investigate two different cost functions to optimize the positioning of the electrodes of a pacemaker on a disease heart. We first consider a cost function which takes into account the delay of a characteristic depolarization time, namely

$$J_1 = t_d - t_{d,target}\,, \tag{9}$$

where t_d represents the first time for which 95 per cent of the whole heart is depolarized:

$$t_d = \inf\{t \geq 0, \quad \text{Volume}(\Omega_t) \geq 0.95\, \text{Volume}(\Omega_H)\}\,,$$

with:

$$\Omega_t = \{x \in \Omega_H, \quad V_m(t, x) > V_s\}\,.$$

and $t_{d,target}$ denotes the same value for the corresponding healthy heart.

The second cost function we consider is based on the behavior of the ECG during the depolarization phase:

$$J_2 = \|D - D_{target}\|_{L^2(0,T)}\,, \tag{10}$$

where D is the so–called first lead of the ECG given by

$$D = \phi_T(L) - \phi_T(R)\,,$$

where L and R are two points of measure located on the left and right arm. As above, D_{target} denotes the same value for the corresponding healthy heart.

3.2 The optimization algorithm

The cost functions J_1 or J_2 previously described are computed after solving a complex set of coupled PDE and ODE with strong three-dimensional effects. Moreover, due to the complexity of the heart geometry, they display a non-smooth behavior with many local minima. For all these reasons, the minimization of J_1 and J_2 is achieved by using evolutionary algorithms and more precisely Genetic Algorithms.

In the present case, a classical real coded Genetic Algorithm is used to optimize the positioning of one or two electrodes of a pacemaker on the internal boundary surface of the heart, also called endocardium. A mapping from the endocardium or a part of it to a simple plane domain, for instance a rectangular domain of $\mathbb{R}^2$, has first been defined in order to simplify the parametric search space.

The selection process used in the Genetic Algorithm is done with a proportionate roulette wheel with respective parts based on the rank of each element in the population. The crossover of two elements is obtained by a barycentric combination with random and independent coefficients in each coordinate whereas the mutation of one element is of non uniform type. Finally, a one-elitism principle is added in order to make sure to keep in the population the best element of the previous generation.

4 Application to a simplified test case

4.1 Description of the test case

The simulations are performed on a simplified geometry which contains ventricles only, see Figure 2.

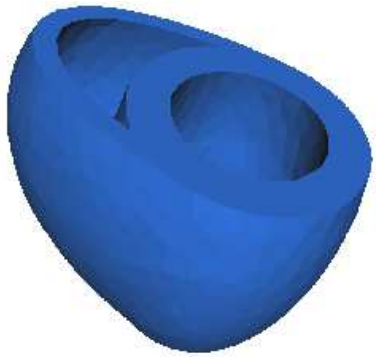

Figure 2: A simplified heart geometry Ω_H.

The domain, closed to a human heart, is analytically defined through its boundary, made of the union of four truncated ellipsoids:

$$(\tfrac{x}{a_{iL}})^2 + (\tfrac{y}{b_{iL}})^2 + (\tfrac{z}{c_{iL}})^2 = 1, \quad (\tfrac{x}{a_L})^2 + (\tfrac{y}{b_L})^2 + (\tfrac{z}{c_L})^2 = 1\,,$$

with $\{a_{iL}\,, b_{iL}\,, c_{iL}\,, a_L\,, b_L\,, c_L\} = \{2.72, 2.72, 5.92, 4, 4, 7.2\}$ cm for the left ventricle internal and external boundary respectively, and

$$(\tfrac{x}{a_{iR}})^2 + (\tfrac{y}{b_{iR}})^2 + (\tfrac{z}{c_{iR}})^2 = 1, \quad (\tfrac{x}{a_R})^2 + (\tfrac{y}{b_R})^2 + (\tfrac{z}{c_R})^2 = 1\,,$$

with $\{a_{iR}\,, b_{iR}\,, c_{iR}\,, a_R\,, b_R\,, c_R\} = \{7.36, 3.36, 6.2, 8, 4, 6.84\}$ cm for the right ventricle. All these ellipsoids are restricted to the half space $z \leq 2.75$.

Note that the pathology considered here, is called a left bundle branch block for which only the right ventricle is initially stimulated. In this case, the electrodes can be placed in the atria and/or in the ventricles. As we only simulate here the ventricles, we seek for the best positioning of the electrodes in the internal surface of the left ventricle.

4.2 Numerical results

We choose the conductivities in (2) and (3) such that the anisotropy of the fibers in the myocardium are taken into account, namely $\sigma_i = \alpha_i^t(I - d_f \otimes d_f) + \alpha_i^l(I - d_f \otimes d_f)$ and $\sigma_e = \alpha_e^t(I - d_f \otimes d_f) + \alpha_e^l(I - d_f \otimes d_f)$, where d_f is the direction of the fibers, I the identity matrix in $\mathbb{R}^3$ and $\alpha_i^t = 5\,10^{-3}\,, \alpha_i^l = 1.5\,10^{-1}\,, \alpha_e^l = 1.\,10^{-1}$ and $\alpha_e^t = 7.5\,10^{-3}$. The parameters in (2)–(7) are choosen as follows: $A_m = C_m = 1$, $\tau_1 = 0.45\,, \tau_2 = 9\,, \tau_3 = 150\,, \tau_4 = 100$, $V_g = 0.13$ and in (8) R_p and C_p are choosen sufficiently small. The intensity of the initial stimulation equals 0.5 mV during 10 ms. The artificial stimulations have the same intensity as the initial stimulation and hold during 40 ms. As we are interested in the depolarization phase only, the final time of computations is actually equal to 100 ms whereas the total duration of depolarization–repolarization process is approximately 300 ms.

The domain Ω_H is discretized with tetrahedra for a total number of nodes equal to 12921. The ionic current is solved by the *cvode*[1] solver, an appropriate solver for stiff nonlinear systems of ODE. The bidomain problem (2)–(6)–(8) is approximated by a piecewise finite elements scheme in space and by a second order backward differences scheme in time. The simulations are done with the C++ library *LifeV*[2].

A number of 30 individuals in taken for the GA population in the case of the optimization of the positioning of one electrode. A number of 10 generations is then needed to achieve a near optimal solution.

We first present the results obtained by using the cost function J_1. In this case the isolated model (2)–(7) is used to simulate the electrical activity of the heart. In the pathological case $J_1 = 73$ ms whereas this delay is reduced to $J_1 = 20.75$ ms when an electrode is placed at the optimal point which actually corresponds to the value

$$(x, y, z) = (3.71, 0.024, -2.03)\,.$$

When we use the cost function J_2, the coupled model (2)–(8) is used and the optimal positionning of the electrode is the following:

$$(x, y, z) = (-1.29, 1.68, -3.69)\,.$$

Note that for this position, the delay in the depolarization process, namely J_1, is far from being optimal as it equals 64.75 ms. These results show that according to the cost function, the optimal positionning of the electrode can be very different.

Figure 3 shows the first lead of ECG, respectively in the reference case, the pathologic case and the pathologic case after an optimization with J_1 and J_2. It is

[1] http://llnl.gov/casc/sundials

[2] http://www.lifev.org/

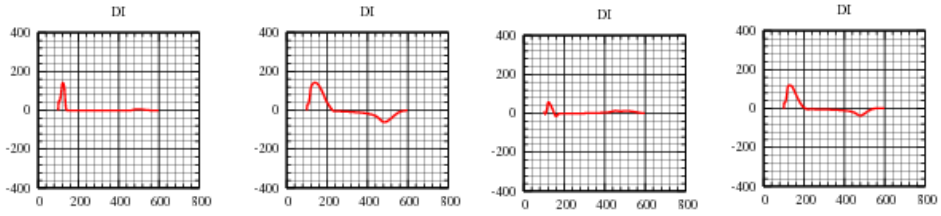

Figure 3: The first lead (DI) of the ECG in the reference case, the pathologic case, the pathologic case with one electrode when J_1 is used and when J_2 is used (from left to right).

natural that the recovery of this particular lead of the ECG is more efficient with the J_2 optimization, in view of its definition in (10). However, the results obtained with the J_1 optimization also lead to a good ECG recovery even if it is based on a different criteria. Moreover, for the corresponding optimal point, all the other leads of the ECG also perform well. It seems to indicate that the J_1 optimization is the more interesting and robust criteria for the optimization of the positioning of the electrodes of a pacemaker.

5 Conclusion

In this article, a simple test case is presented to show how optimization can improve the placement of the electrodes of a pacemaker. A robust cost function based on the depolarization delay of the disease heart is in particular introduced. The obtained result show how a rather good ECG can be recovered, even with only one electrode, after an optimization of its positioning.

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